In: Proceedings of Institute of Electrical and Electronic Engineers, NECEC '90, Newfoundland Electrical and Computer Engineering Conference, "An Electrical Engineering Day". St. John's, NF, May 4, 1990, Session 3A.
C-CORE Publications No. 90-5.

MEASUREMENT OF OCEAN WAVE SPECTRA USING A SHIP-MOUNTED HF RADAR

R. Howell and J. Walsh

Centre for Cold Ocean Resources Engineering
Memorial University of Newfoundland
St. John's, Newfoundland, Canada, A1B 3X5

Abstract

A data interpretation technique is presented and tested to extract the nondirectional ocean wave-height spectrum from the echo return of a ship-mounted HF radar employing an omnidirectional receive/transmit antenna. Tests of this technique at sea shows that there is a high degree of correlation between its wave spectra estimates and that of a WAVE-TRACK buoy.

I Introduction

A promising alternative to *in situ* methods (e.g., wave buoys) for measuring ocean wave spectra is HF ground wave radar. Land-based narrow-beam systems have already demonstrated their feasibility for this task [1].

In this paper, an algorithm is described which may be used to extract the nondirectional wave-height spectrum from the backscatter return of a stationary ship- or rig-mounted radar employing an omnidirectional antenna (e.g., a single monopole) for both transmission and reception. The basis of this algorithm is the inversion of the integral equation representing the omnidirectional second-order radar cross section of the ocean surface at HF. The inversion technique used here may be thought of as an adaption of that used in [1] to solve the narrow-beam equation.

To test the effectiveness of the algorithm, its estimates are compared to those of a WAVE-TRACK buoy using data collected onboard CFAV Quest during 1988.

II Radar Cross Section of the Ocean Surface

The distribution of wave energy as a function of frequency, f, and direction, θ, may be represented by the spectral density function $E(f,\theta)$. By definition, $E(f,\theta)$ is the ocean wave spectrum. A common and very useful representation for $E(f,\theta)$ is to expand it into a Fourier series versus angle. Such an expansion has the form

$$E(f,\theta) = \sum_{n=0}^{\infty} a_n(f)\cos(n\theta) + b_n(f)\sin(n\theta) \quad (1)$$

where $b_0(f) = 0$.

Due to the dispersive properties of the ocean surface, the frequency of an ocean gravity wave and its wavenumber, k, is related by

$$f = \sqrt{gk\tanh(kd)}/(2\pi) \quad (2)$$

where g is the acceleration due to gravity and d is water depth.

Shown in Fig. 1 is an example of an omnidirectional 25.4 MHz radar Doppler spectrum. It may be observed that the dominant feature of this spectrum are the two well defined peaks symmetrically placed about zero Doppler at about $\pm.5$Hz. The physical mechanism responsible for these "first-order" peaks is that of coherent Bragg scatter [2]. In this diffraction-grating mechanism, resonant backscatter will arise from those ocean waves whose wavelength is exactly one-half the radar wavelength. Those ocean waves which satisfy this geometry are referred to as the "Bragg waves". Based upon the dispersion relationship (2), the Doppler shift, f_B, of the first-order peaks in the absence of currents may be given as

$$f_B = \sqrt{2gk_o\tanh(k_o d)}/(2\pi) \quad (3)$$

where k_o is the radar wavenumber.

For narrow-beam systems, Walsh and Howell [3] have derived the following expression for the total cross section power of the first-order peaks:

$$R_{m'} = TE(f_B, (1+m')\frac{\pi}{2} + \phi) \quad (4)$$

where the sign of the subscript m' identifies whether R, the energy of the peak, is for the pos-

itive or negative Doppler half of the radar spectrum and ϕ is the look direction of the narrow-beam radar. The quantity T is defined as

$$T = 8k_o^3(\pi^2 f_B^2 + gk_o^2 d\,\mathrm{sech}^2(2k_o d))/(\pi f_B) \quad (5)$$

The omnidirectional first-order cross section may be obtained from the narrow-beam equation by integrating it over ϕ. The resultant expression for the omnidirectional first-order power, $R^o_{m'}$, is

$$R^o_{m'} = \int_0^{2\pi} R_{m'}\, d\phi = 2\pi T a_0(f_B) \quad (6)$$

From Fig. 1 it may be observed that there is a sideband continuum surrounding the first-order peaks. It was determined from [3] that second-order interactions (i.e., wave-wave interactions and second-order Bragg scatter) form the dominant contribution to this continuum. For this component of the radar cross section, the incident radio wave interacts with pairs of ocean waves. One of these ocean waves will be of roughly the same length as the short Bragg waves. The frequency and wavenumber of these waves shall be represented by f' and k' respectively. The other scattering wave corresponds to those long ocean waves containing the bulk of the ocean spectral energy. the frequency and wavenumber of this set of waves shall be represented by f and k respectively.

For narrow-beam systems, the second-order cross section, σ_2, as a function of Doppler frequency, f_d, has been derived by Walsh and Howell [3] as

$$\sigma_2(\omega_d) = \int_{-\pi}^{\pi} CE(f, \alpha + \phi)E(f', \alpha' + \phi)\, d\theta \quad (7)$$

where

$$f_d - mf + m'f' = 0, \quad f' \geq f \quad (8)$$

$$\alpha = \theta + (1 - m)\pi/2 \quad (9)$$

$$\alpha' = \theta' + (1 - m')\pi/2 \quad (10)$$

$$k' = \sqrt{k^2 + 4kk_o\cos(\theta) + 4k_o^2} \quad (11)$$

$$\theta' = \arcsin(k\sin(\theta)/k') + \pi \quad (12)$$

$$m' = \mathrm{sgn}(f_d) \quad (13)$$

$$m = \begin{cases} m' & \text{for } |f_d| > f_B \\ -m' & \text{for } 0 < |f_d| < f_B \end{cases} \quad (14)$$

The quantity C represents the kernel function of the second-order integral (see [3]). In evaluating (7) the constraint equation (8) must be solved for each θ to determine the value of f that satisfies it.

Like the first-order case, the omnidirectional second-order cross section may be determined from the narrow-beam equation by integrating it over ϕ, i.e.,

$$\begin{aligned}\sigma_2^o(f_d) &= \int_0^{2\pi} \sigma_2(f_d)\, d\phi \\ &= 2\pi \int_0^{\pi} d\theta\, C \sum_{n=0}^{\infty} H L^n \cos(n(\theta - \theta')) \\ &\quad \cdot [a_n(f)a_n(f') + b_n(f)b_n(f')] \end{aligned} \quad (15)$$

where

$$L = mm' \quad (16)$$

$$H = \begin{cases} 2 & \text{for } n = 0 \\ 1 & \text{otherwise} \end{cases} \quad (17)$$

In writing (15), use has been made of the fact that both C and the constraint equation (8) are even functions of θ.

Equation (15) represents a nonlinear, two-dimensional integral equation of the first kind. By inverting this equation, ocean spectral information may be obtained from the radar spectrum.

III Linearization of the Integral Equation

Although (15) is nonlinear, it may be easily linearized to remove all f' ocean spectral terms. This is accomplished by dividing the second-order return by the first-order power and taking advantage of the fact that the short scattering wave f' is of approximately the same length as the Bragg waves.

As both f' and the Bragg waves correspond to short wavelengths, they will generally lie in the saturated region of the ocean wave spectrum. At this equilibrium level where no further growth is possible, the wave spectrum depends upon frequency as f^{-5} [4]. Also, these short waves will tend to share the same directional distribution. Hence, wave components of frequency f' may be related to those of the Bragg waves as

$$E(f', \alpha') = E(f_B, \alpha')[f_B/f']^5 \quad (18)$$

or equivalently as

$$\{a_n(f'), b_n(f')\} = \{a_n(f_B), b_n(f_B)\}[f_B/f']^5 \quad (19)$$

Substituting (19) into (15) and dividing each half of the radar spectrum by its value of $R_{m'}$, the resulting linearized second-order cross section, $\sigma_{2L}(f_d)$, may be written as

$$\sigma^o_{2L}(f_d) = \frac{\sigma^o_2(f_d)}{R^o_{m'}}$$
$$= \int_0^\pi d\theta\, C' \sum_{n=0}^{\infty} H L^n \cos(n(\theta - \theta'))$$
$$\cdot \left[a_n(f) \frac{a_n(f_B)}{a_0(f_B)} + b_n(f) \frac{b_n(f_B)}{a_0(f_B)} \right] \quad (20)$$

where

$$C' = C(f_B/f')^5/T \quad (21)$$

Not only does this approach linearizes the equation by removing the f' ocean spectral terms, it has the elegant advantage of normalizing it as well. That is, all transmissions factors affecting the radar spectrum are divided out when the spectrum is divided by the first-order power. Hence, HF radars do not require any calibration of its wave measurement to take into account the path gains and losses of the received radar signal.

IV Solution of the Integral Equation

The solution method to be applied to the linearized equation (20) is the same as that used in [1] to solve the narrow-beam integral equation. Like [1], the Fourier series expansion for the wave spectrum is truncated after $n = 2$ and the integral equation is approximated for a similar range of Doppler as a matrix equation of the form $Ax = b$. The solution to this equation is calculated by pseudo-inverting the kernel matrix A using a singular value decomposition [5].

However, the technique used here differs from that of [1] in that the matrix equation uses a different set of variables. For a narrow-beam system, these variables are the individual Fourier coefficients $b_2(f)$, $b_1(f)$, $a_0(f)$, $a_1(f)$, and $a_2(f)$. For an omnidirectional radar these variables are $a_o(f)$, $[a_1(f)a_1(f_B) + b_1(f)b_1(f_B)]/a_0(f_B)$, and $[a_2(f)a_2(f_B) + b_2(f)b_2(f_B)]/a_0(f_B)$. Obviously, the last two variables are of little use for interpreting useful wave information. This reflects the fact that an omnidirectional radar cannot provide directional information. In order to obtain directional information, a larger aperture receive array is required.

Although no directional information may be obtained, an omnidirectional radar may still measure $a_0(f)$ which is directly related to the nondirectional waveheight spectrum $E(f)$ as

$$E(f) = a_0(f)\pi/2 \quad (22)$$

By and large, the most sought after information for wave studies is $E(f)$ as the most important ocean spectral parameters (e.g., significant waveheight) may be determined from it. Indeed, most wave buoys have been designed to measure $E(f)$ only.

V Test Results and Conclusions

Shown in Figs. 2 and 3 is a set of wave measurements taken over a 3 hour period on May 3, 1988 by a WAVE-TRACK buoy and a 25.4 MHz radar mounted onboard CFAV Quest. The WAVE-TRACK's estimate for $E(f)$ ranged from .03 to .3 Hz while the radar's estimate ranged from .09 to .25 Hz. All measurements were collected while the ship was drifting.

The buoy estimate for $E(f)$ shown in Fig. 1 was collected during the first 20 minutes of the experiment with the 20 minute radar observation taken 20 minutes after the buoy had finished. Although there is a 40 minute temporal difference between the midpoint of each operation period, wind conditions remained stable at about 20 knots. For this case, there is little difference between the wave spectrum estimates provided by each sensor.

The buoy estimate for $E(f)$ shown in Fig. 2 was collected exactly 3 hours after the first buoy measurement of Fig. 1 and 1 hour after the radar finished collecting its data for the second time. For this case, there is a 80 minute temporal difference between the midpoint of each operation period. Also, the wind speed increased to 30 knots shortly after the measurements of Fig. 1 were taken. Hence, wave conditions should no longer remain steady as they did for Fig. 1, but be in a state of growth.

This expected growth of the wave spectrum is reflected in the differences between the $E(f)$ estimates of Fig. 2. In fact, the radar estimate for $E(f)$ is entirely consistent with what the intermediate spectrum should look like as the spectrum of Fig. 1 grows over a 3 hour period to that of the buoy's measurement of Fig. 2.

In summary, even with the temporal differences of this experiment, there is high degree of correlation between $E(f)$ estimates produced by the omnidirectional inversion algorithm and a WAVE-TRACK buoy. With these positive re-

sults, the basic feasibility of HF radar measurement of sea state on the open ocean has been established. Also, the Walsh and Howell cross section model has been further validated.

Work will now proceed to deploy directional arrays onboard ships to further test the inversion algorithm and to develop a facility to collect directional information as well.

Acknowledgements

The authors wish to thank the Defence Research Establishment Atlantic (DREA) for the use of their vessel and for providing the WAVE-TRACK buoy. This work was supported by a NSERC strategic grant to John Walsh.

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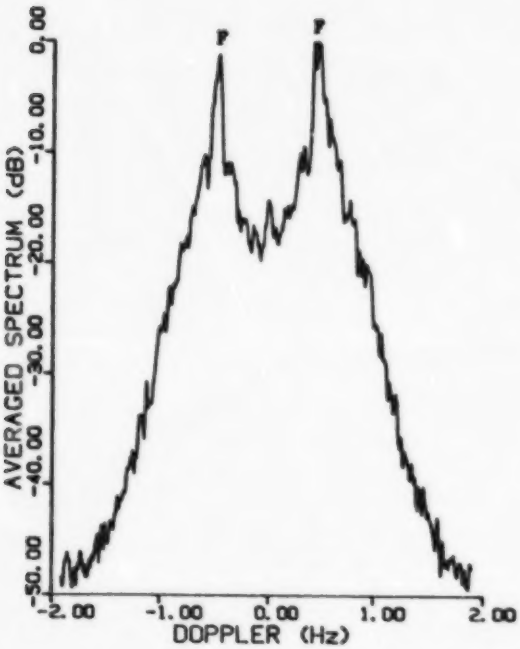

Figure 1: Example of a radar Doppler spectrum recorded by a 25.4 MHz omnidirectional system. First order peaks are designated by F.

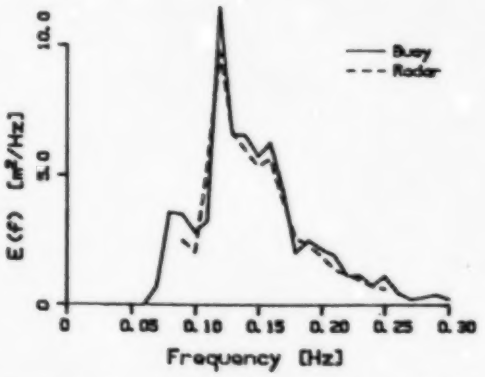

Figure 2: Comparison of ground wave radar and WAVE-TRACK $E(f)$ estimates for the first set of experiment observations.

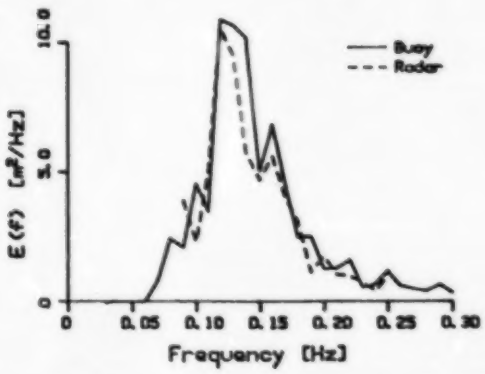

Figure 3: Comparison of ground wave radar and WAVE-TRACK $E(f)$ estimates for the second set of experiment observations.